The interaction of transverse electromagnetic plane waves and a moving ionizing shock wave in the presence of a magnetic field

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(Received 2 May 1963 and in revised form 9 September 1963)

Propagation of a transverse electromagnetic wave along a d.c. magnetic field and interaction with a moving shock wave is investigated. The direction of propagation is normal to the shock front. Solutions to electromagnetic fields, gas velocities and Doppler shifts in frequency are found in both the ionized and un-ionized gases. A frequency is obtained at which electromagnetic reflexion from the shock front is minimized. In the ionized gas behind the shock front a fast and a slow electromagnetic wave result. By adjusting the shock velocity the frequency of the slow wave can be either raised or lowered. This frequency change, however, is not the Doppler shift and, consequently, can be made much larger than the Doppler shifts encountered at non-relativistic velocities. The slow wave attenuates much less than the ordinary fast wave, and applications to diagnostics and communications through plasmas and laser-beam frequency multiplication may be possible.

1. Introduction

The interaction of electromagnetic waves and moving ionizing shock waves is of interest in plasma diagnostics, ionospheric physics, and may have application to problems of communication through plasma sheaths. Interactions with stationary discontinuities have been discussed by Kahalas (1963) and Ullah & Kahalas (1963), and with moving shocks in the absence of a magnetic field by many investigators interested in diagnostic techniques (Chang 1961; Brandewie 1963).

The problem of this interaction with a moving surface of an electron gas (with the gas moving with the same velocity as the free surface) has been treated by Lampert (1956), Fainberg & Tkalich (1959) and Kurilko & Miroshnichenko (1962).

We treat here the problem of the interaction of a plane T.E.M. wave propagating in a direction normal to a plane shock wave in the presence of a magnetic field. Two characteristic velocities then appear, the shock velocity and the velocity of the gas behind the shock. The gas in front of the shock is assumed to be un-ionized, and the gas behind the shock ionized and electrically conducting. A uniform steady magnetic field B_0 is applied in the direction of propagation (normal to the shock wave). Since the applied magnetic field is normal to the shock front, the ordinary Rankine–Hugoniot relations for an ionizing shock apply, and hydromagnetic shock effects need not be considered.

The shock itself may be an oblique shock (with respect to the fluid velocities). The un-ionized gas in front of the shock is completely uncoupled from the electromagnetic wave, and behind the shock only the normal component of velocity couples into the relevant equations. We do assume, however, that the shock is normal with respect to the applied magnetic field and direction of propagation. Figure 1 illustrates the physical situation. The primed co-ordinate system is taken to be at rest with respect to the shock front. In this co-ordinate system the normal component of the approaching gas is V_s, and the normal component of the ionized receding gas is V_g which is, of course, less than V_s.

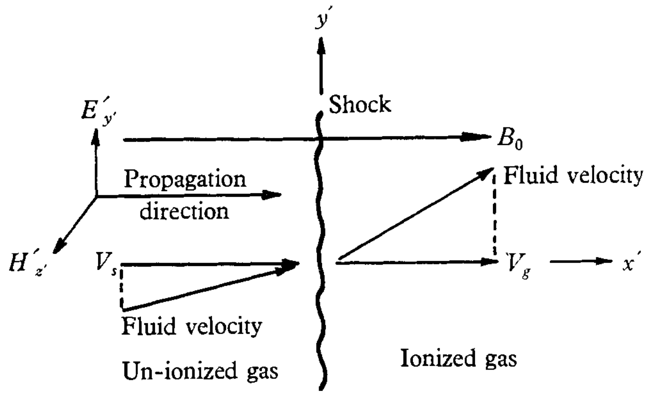

FIGURE 1. The general shock frame of reference showing relative gas velocities.

One particular application of the solution is to the coaxial shock tube used for diagnostic purposes (Chang 1961; Brandewie 1963). Figure 2 shows such a tube. In the tube the shock is normal to the fluid velocities. The velocity of the shock wave is V_s in the negative x-direction. The absolute velocity of the gas is $(V_s - V_g)$ in the negative x-direction. The magnetic field is applied in the x-direction which is the direction of propagation. The problem is analysed in terms of this diagram, although the results are valid for the more general situation shown in figure 1. A T.E.M. wave is initiated in the laboratory frame of reference and propagates in the x-direction along the tube. At the moving shock surface, the wave is partially reflected and partially transmitted, with frequency shifts and splitting. In the shock frame of reference the angular frequency is a constant, ω', but in the laboratory frame, where measurements are usually made, the frequency of the reflected wave undergoes a Doppler shift, and the transmitted waves may have frequencies entirely different from that of the incident wave. Of particular interest are the reflexion coefficient and transmission coefficient which are calculated in both the shock and laboratory frames of reference.

The ionized gas is assumed to be a continuum with a scalar conductivity, although the displacement current is retained throughout. While this continuum assumption limits the applicability of the solution, many real physical situations are concerned with gases of sufficient density to allow this approximation. Any further refinement of the gas model would tend to obscure the simple results of the calculations which should hold at least qualitatively for more exact models.

2. Formulation of the problem

In figure 2 a shock wave moves in the negative x-direction with a speed V_s. An axial magnetic field B_0 is applied in the x-direction and an electromagnetic (T.E.M.) wave propagating in the x-direction is initiated. The gas behind the shock wave moves with an absolute velocity V_0 which is less than V_s. Relative to an observer at rest with respect to the shock front the ionized gas moves in the x-direction to the right with a velocity $V_g = (V_s - V_0)$.

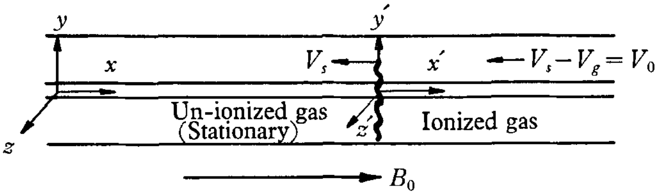

FIGURE 2. The coaxial shock tube. The laboratory frame co-ordinate system (x, y) is fixed with respect to the shock tube. The shock frame or rest frame of reference (x', y') is fixed with respect to the shock front. The shock moves to the left (negative x-direction) with velocity V_s, and the absolute velocity of the ionized gas is V_0 in the negative x-direction.

To begin the solution Maxwell's equations and the fluid equations of motion are applied by an observer in the rest (or shock) frame of reference. These equations are solved simultaneously in the rest frame and subjected to the appropriate boundary conditions which are discussed later. In the solution primed quantities refer to shock- or rest-frame quantities and unprimed quantities to the laboratory frame. Maxwell's 'curl' equations are

$$\nabla' \times \mathbf{E}' = -\mu_0 \partial \mathbf{H}'/\partial t', \tag{1}$$

$$\nabla' \times \mathbf{H}' = \mathbf{J}' + \epsilon_0 \partial \mathbf{E}'/\partial t'. \tag{2}$$

Here it is assumed that the gas has the permeability μ_0 and permittivity ϵ_0 of free space. The appropriate form of Ohm's law for the ionized gas is

$$\mathbf{J}' = \sigma(\mathbf{E}' + \mu_0 \mathbf{V}' \times \mathbf{H}'), \tag{3}$$

where σ and $\mathbf{J}'$ are the electrical conductivity and current density respectively. $\mathbf{V}'$ is the velocity of the gas with respect to the shock frame. The equation of motion for the ionized gas is

$$\rho'[\partial \mathbf{V}'/\partial t' + (\mathbf{V}'.\nabla')\mathbf{V}'] = -\nabla' P' + \mu_0 \mathbf{J}' \times \mathbf{H}', \tag{4}$$

where viscosity is neglected and ρ' and P' are the mass density and pressure respectively. Assuming propagation in the x'-direction, all x' and t' variations can be replaced by $\exp\{i(\omega' t' - \gamma' x')\}$ where ω' and γ' are the angular frequency and propagation constant respectively. Considering only transverse waves, $\partial/\partial z' = \partial/\partial y' = 0$ and (1) to (4) simplify greatly under the assumption that the incident electromagnetic wave is comprised of an E'_{y0} and H'_{z0}. The zero subscript refers to field quantities in the un-ionized gas. Equations (1), (2) and (4) combined with Ohm's law can be written after linearization as

$$\gamma' E_y'^* = \omega' \mu_0 H'^*, \tag{5}$$

$$i\gamma' H_z'^* = \sigma E_y'^* + i\omega' \epsilon_0 E_y'^* - V_g \sigma \mu_0 H_z'^* + \sigma V_z'^* B_0, \tag{6}$$

$$i\rho'(\omega' - \gamma' V_g) V_z'^* = -\sigma B_0 (E_y'^* - V_g \mu_0 H_z'^* + B_0 V_z'^*), \tag{7}$$

and Ohm's law may be written explicitly as

$$J_y'^* = \sigma(E_y'^* - V_g\mu_0 H_z'^* + B_0 V_z'^*), \tag{8}$$

where the starred quantities are phasors. The other components of the equations and the other Maxwell equations are uncoupled and are not considered here. From (5), (6) and (7) the dispersion equation for transverse oscillations is obtained as

$$\begin{vmatrix} \mu_0\gamma' & -\omega'\mu_0 & 0 \\ \mu_0\sigma + i\omega'\epsilon_0\mu_0 & -(i\gamma' + V_g\sigma\mu_0) & \sigma B_0 \\ \mu_0\sigma B_0 & -V_g\sigma\mu_0 B_0 & \sigma B_0^2 + i\rho'(\omega' - \gamma' V_g) \end{vmatrix} = 0. \tag{9}$$

Expanded, the determinant becomes

$$\gamma'^3 - \left[\frac{\omega'}{V_g} + i\frac{V_g}{\eta}\left(1 - \frac{a^2}{V_g^2}\right)\right]\gamma'^2 - \left[\left(\frac{\omega'}{c}\right)^2 - \frac{2i\omega'}{\eta}\right]\gamma' + \frac{\omega'}{V_g}\left[\frac{\omega'^2}{c^2} + i\frac{\omega'}{\eta}\left(1 + \frac{a^2}{c^2}\right)\right] = 0, \tag{10}$$

where the diffusivity $\eta = (\sigma\mu_0)^{-1}$ and the Alfvèn velocity $a = (B_0^2/\rho'\mu_0)^{\frac{1}{2}}$ have been introduced. c is the velocity of light and is, of course, related to the permeability and permittivity by $c^2 = (\mu_0\epsilon_0)^{-1}$. Another equation of interest is obtained from (1) and (6) in terms of V_z' as

$$V_z' = -\frac{E_y'}{B_0}\left[1 - \frac{\gamma' V_g}{\omega'} + i\eta\left(\frac{\omega'}{c^2} - \frac{\gamma'^2}{\omega'}\right)\right]. \tag{11}$$

Equation (10) has three roots for the propagation constant γ'. Two roots represent fast and slow forward waves (denoted as γ_2' and γ_1') and the third root represents a backward wave, assumed to be null. This assumption is equivalent to neglecting the reflexions from the far right end of the shock tube. The electric and magnetic field solutions in the ionized gas are then (as measured by an observer in the shock frame)

$$E_y' = C'\exp\{i(\omega' t' - \gamma_1' x')\} + D'\exp\{i(\omega' t' - \gamma_2' x')\}, \tag{12}$$

$$H_z' = (\gamma_1'/\mu_0\omega')\,C'\exp\{i(\omega' t' - \gamma_1' x')\} + (\gamma_2'/\mu_0\omega')\,D'\exp\{i(\omega' t' - \gamma_2' x')\}. \tag{13}$$

Upon substitution of E_y' from (12) into (11) there results

$$V_z' = -\frac{1}{B_0}\left\{\left[1 - \frac{V_g\gamma_1'}{\omega'} + i\eta\left(\frac{\omega'}{c^2} - \frac{\gamma_1'^2}{\omega'}\right)\right] C'\exp\{i(\omega' t' - \gamma_1' x')\} + \left[1 - \frac{V_g\gamma_2'}{\omega'} + i\eta\left(\frac{\omega'}{c^2} - \frac{\gamma_2'^2}{\omega'}\right)\right] D'\exp\{i(\omega' t' - \gamma_2' x')\}\right\}. \tag{14}$$

In the un-ionized gas before the shock front the fields (as measured by an observer in the shock frame) are

$$E_{y0}' = A'\exp\{i\omega'(t' - x'/c)\} + B'\exp\{i\omega(t' + x'/c)\}, \tag{15}$$

$$H_{z0}' = (\epsilon_0/\mu_0)^{\frac{1}{2}}\,[A'\exp\{i\omega'(t' - x'/c)\} - B'\exp\{i\omega'(t' + x'/c)\}]. \tag{16}$$

In the above expressions for the fields, A' is the amplitude of the incident wave and B', C' and D' must be obtained from the boundary conditions. The reflexion and transmission coefficients as measured in the shock frame can then be expressed in terms of these constants.

3. Boundary conditions

To the observer in the shock frame,

$$E'_y(x' = 0) = E'_{y0}(x' = 0) \quad \text{and} \quad H'_z(x' = 0) = H'_{z0}(x' = 0),$$

which expresses the continuity of the tangential fields across the shock front. These are the well-known relations that apply at a boundary between adjoining media and will yield two independent equations for A', B', C' and D'. However, three equations are needed in order to determine B', C' and D' in terms of the incident amplitude A'. The third condition is obtained from the continuity of the tangential component of velocity induced by the electromagnetic interaction. At the shock front $V'_z(x' = 0) = V'_{z0}(x' = 0)$. But V'_{z0} is everywhere zero since there is no electromagnetic coupling in the un-ionized gas, and no effects behind the shock can propagate upstream. Although H'_z is in the plane of the shock and gives rise to a magnetohydrodynamic shock effect, H'_z and V'_z must be continuous unless the conductivity is infinite. A current sheet would have to exist in the shock front in order to sustain any discontinuity and such sheets can occur only in gases with essentially infinite conductivity. The magnetohydrodynamic shock interaction is neglected since it is a second-order effect. This procedure is standard and is discussed in the references. Therefore, $V'_z(x' = 0)$ is taken as zero.

This condition in equation (11) yields

$$C' = \frac{1 - V_g\gamma'_2/\omega' + i\eta(\omega'/c^2 - \gamma_2'^2/\omega')}{1 - V_g\gamma'_1/\omega' + i\eta(\omega'/c^2 - \gamma_1'^2/\omega')} D', \tag{17}$$

and by continuity of the fields

$$C' + D' = A' + B', \tag{18}$$

$$c(\gamma'_1 C' + \gamma'_2 D') = \omega'(A' - B'). \tag{19}$$

The coefficients B', C' and D' are then found in terms of A' and

$$B' = \frac{V_g - c + i\eta(\gamma'_1 + \gamma'_2 - \gamma'_1\gamma'_2 c/\omega' - \omega'/c)}{V_g + c + i\eta(\gamma'_1 + \gamma'_2 + \gamma'_1\gamma'_2 c/\omega' - \omega'/c)} A', \tag{20}$$

$$C' = \frac{2\omega'[1 - V_g\gamma'_2/\omega' + i\eta(\omega'/c^2 - \gamma_2'^2/\omega')]}{(\gamma'_1 - \gamma'_2)[V_g + c + i\eta(\gamma'_1 + \gamma'_2 + \gamma'_1\gamma'_2 c/\omega' + \omega'/c)]} A', \tag{21}$$

$$D' = \frac{-2\omega'[1 - V_g\gamma'_1/\omega' + i\eta(\omega'/c^2 - \gamma_1'^2/\omega')]}{(\gamma'_1 - \gamma'_2)[V_g + c + i\eta(\gamma'_1 + \gamma'_2 + \gamma'_1\gamma'_2 c/\omega' + \omega'/c)]} A'. \tag{22}$$

4. Fields in the laboratory frame

The magnitude of the various waves can now be transformed into the laboratory co-ordinate frame. The appropriate transformations

$$\beta E_y = (\mathbf{E}' - \beta \mathbf{V} \times \mathbf{B})_y = E'_y - \beta\mu_0 V_s H'_z \tag{23}$$

and

$$\beta H_z = (\mathbf{H}' + \beta\epsilon_0 \mathbf{V} \times \mathbf{E})_z = H'_z - \beta\epsilon_0 V_s E'_y, \tag{24}$$

where $\beta = (1 - V_s^2/c^2)^{-\frac{1}{2}}$ and $\mathbf{V}$ is the velocity of the shock frame with respect to the laboratory frame. Because $|V_s/c| \ll 1$, the amplitudes of the electromagnetic waves in the un-ionized gas are essentially the same in the laboratory and shock frames by virtue of (23) and (24). To complete the solution in the un-ionized gas the laboratory-frame frequency and propagation constant must be determined.

The fields in the un-ionized gas seen by the laboratory observer are of the form

$$E_{y0} = A' \exp\{i[\omega_i t - \gamma_i(x-x_0)]\} + B' \exp\{i[\omega_r t + \gamma_r(x-x_0)]\}, \tag{25}$$

and

$$H_{z0} = (\epsilon_0/\mu_0)^{\frac{1}{2}} [A' \exp\{i[\omega_i t - \gamma_i(x-x_0)]\} - B' \exp\{i[\omega_r t + \gamma_r(x-x_0)]\}], \tag{26}$$

where ω_i and ω_r are the angular frequencies of the incident and reflected waves respectively. x_0 is the position of the shock wave at some arbitrary initial time from which time t (in the laboratory frame) is reckoned. The arguments of the exponentials in (15) can be transformed by the Lorentz transformations of x' and t', $x' = \beta[(x-x_0)+V_s t]$ and $t' = \beta[t + V_s(x-x_0)/c^2]$. This operation yields

$$\omega' t' - \gamma' x' = \beta(\omega' - \gamma' V_s)t - \beta\gamma'(x-x_0)(1 - V_s/c) \tag{27}$$

for the incident wave and

$$\omega' t' + \gamma' x' = \beta(\omega' + \gamma' V_s)t + \beta\gamma'(x-x_0)(1 + V_s/c) \tag{28}$$

for the reflected wave. The transformed arguments of (27) and (28) should equal the corresponding arguments of (25). Therefore

$$\gamma_r = \beta\gamma'(1+V_s/c), \quad \gamma_i = \beta\gamma'(1-V_s/c), \tag{29}$$

$$\omega_i = \beta\omega'(1-V_s/c) = \beta(\omega' - \gamma' V_s), \tag{30}$$

$$\omega_r = \beta\omega'(1+V_s/c) = \beta(\omega' + \gamma' V_s), \tag{31}$$

and hence the reflected wave frequency is

$$\omega_r = \omega_i(1+V_s/c)/(1-V_s/c) \approx \omega_i(1+2V_s/c) \tag{32}$$

in terms of the incident frequency.

Finally, then, the fields in the un-ionized gas written in laboratory frame co-ordinates are

$$E_{y0} = A' \exp[i\omega_i\{t - (x-x_0)/c\}] + B' \exp[i\omega_i(1+2V_s/c)\{t + (x-x_0)/c\}], \tag{33}$$

$$H_{z0} = (\epsilon_0/\mu_0)^{\frac{1}{2}} \{A' \exp[i\omega_i\{t-(x-x_0)/c\}] \\ - B' \exp[i\omega_i(1+2V_s/c)\{t+(x-x_0)/c\}]\}, \tag{34}$$

giving rise to the usual Doppler shift.

To find the frequency, propagation constants and attenuation in the ionized gas as seen by an observer in the laboratory frame, the Lorentz transformations are applied to the terms $i(\omega' t' - \gamma_k' x')$ where $k = 1$ or 2, corresponding to the two forward waves in the ionized gas. The γ_k are complex. Let the real and imaginary parts of γ_k be denoted by $\mathrm{Re}\,\gamma_k$ and $\mathrm{Im}\,\gamma_k$ respectively. Then

$$(\omega' t' - \gamma_k' x') = \beta(\omega' - \gamma_k' V_s)t - \beta\gamma_k'(x-x_0) + \beta\omega' V_s(x-x_0)/c^2. \tag{35}$$

This last result can be rewritten as

$$\beta(\omega' - \mathrm{Re}\,\gamma_k' V_s)t - \beta\,\mathrm{Re}\,\gamma_k'(x-x_0) - i\beta[\mathrm{Im}\,\gamma_k' V_s t + \mathrm{Im}\,\gamma_k'(x-x_0)] + \beta\omega' V_s(x-x_0)/c^2.$$

Hence, we recognize that the frequency of the transmitted waves seen by the laboratory-frame observer are $\omega_{Tk} = \beta(\omega' - \mathrm{Re}\,\gamma_k' V_s)$ and the propagation constant is $\beta(\mathrm{Re}\,\gamma_k' - \omega' V_s/c^2)$. In addition, there is attenuation, which is to be expected, for at a point in the ionized gas, the laboratory observer sees the shock wave moving and producing more fluid which attenuates the fields. (The imaginary parts of γ_k and γ_k' will be negative.)

The wave transmitted through the shock front splits into two waves of different frequencies (as measured in the laboratory frame). By (30) then the frequencies and propagation constants of the transmitted waves become

$$\omega_{T1} = [\omega_i/(1 - V_s/c)] - \beta\,\mathrm{Re}\,\gamma_1' V_s, \quad \mathrm{Re}\,\gamma_1 = \mathrm{Re}\,\beta\gamma_1' - \beta\omega' V_s/c^2, \tag{36}$$

$$\omega_{T2} = [\omega_i/(1 - V_s/c)] - \beta\,\mathrm{Re}\,\gamma_2' V_s, \quad \mathrm{Re}\,\gamma_2 = \mathrm{Re}\,\beta\gamma_2' - \beta\omega' V_s/c^2. \tag{37}$$

The fields in the ionized gas (as measured in the laboratory frame) can be written then

$$\begin{aligned} E_y = {} & C_1 \exp\{i[\omega_{T1} t - \beta(\mathrm{Re}\,\gamma_1' - \omega' V_s/c^2)(x - x_0)] + \beta\,\mathrm{Im}\,\gamma_1'[V_s t + (x - x_0)]\} \\ & + D_1 \exp\{i[\omega_{T2} t - \beta(\mathrm{Re}\,\gamma_2' - \omega' V_s/c^2)(x - x_0)] + \beta\,\mathrm{Im}\,\gamma_2'[V_s t + (x - x_0)]\}. \end{aligned} \tag{38}$$

$$\begin{aligned} H_z = {} & C_2 \exp\{i[\omega_{T1} t - \beta(\mathrm{Re}\,\gamma_1' - \omega' V_s/c^2)(x - x_0)] + \beta\,\mathrm{Im}\,\gamma_1'[V_s t + (x - x_0)]\} \\ & + D_2 \exp\{i[\omega_{T2} t - \beta(\mathrm{Re}\,\gamma_2' - \omega' V_s/c^2)(x - x_0)] + \beta\,\mathrm{Im}\,\gamma_2'[V_s t + (x - x_0)]\}. \end{aligned} \tag{39}$$

It should be remembered that the time t is reckoned from the instant that the shock wave is located at position x_0 in the laboratory frame.

The phase velocities of the two transmitted waves may be calculated in the shock frame. The phase velocities are simply ω'/γ_1' and ω'/γ_2' in the shock frame. As will be shown, in a frame of reference at rest with respect to the ionized gas, the γ_1' wave is a non-propagating wave, and the γ_2' wave remains a fast forward wave.

In the laboratory frame, the fast wave remains a fast forward wave, but obviously the slow wave (standing in the gas) must look like a backward wave travelling with the gas in the negative x-direction. This is indeed the case and ω_{T1} will always be a negative number, while γ_1 is a positive number. This implies that the ω_{T1} wave is a backward wave generated in the ionized gas as it is formed by the moving shock (Lampert 1956). The constants C_1, C_2, D_1 and D_2 are obtained from (23) and (24). By equating coefficients there results:

$$C_1 = C', \tag{40}$$

$$D_1 = D', \tag{41}$$

$$C_2 = \left(\frac{\gamma_1'}{\mu_0 \omega'}\right) C', \tag{42}$$

$$D_2 = \left(\frac{\gamma_2'}{\mu_0 \omega'}\right) D'. \tag{43}$$

5. Solution to the dispersion equation

The solution to equation (10) is easily found when $|\omega'/V_g| \gg |V_g/\eta|$ which would be the case for most physical problems. The application of Newton's iterative method yields

$$\gamma_1' \cong (\omega'/V_g) - i(a^2/\eta V_g) \tag{44}$$

for the largest root with a phase velocity of $V_{p1}' = V_g$ which indicates that the γ_1' wave is a slow wave propagating in the positive x'-direction and is actually exactly a standing wave in the ionized gas frame of reference. An additional iteration using Newton's method reveals a correction to this term of the form $i(V_g^5/\eta^3\omega'^2)$ which is negligible compared to $a^2/\eta V_g$. The other two roots are obtained by dividing (10) by $(\gamma' - \gamma_1')$. The resulting roots are equal but opposite in sign. The one representing the forward travelling wave, of interest here, is

$$\gamma_2' \cong (1-i)(\omega'/2\eta)^{\frac{1}{2}} \tag{45}$$

with a phase velocity $V_{p2}' = (2\omega'\eta)^{\frac{1}{2}}$ which is a fast wave propagating in the positive x'-direction. Using (44) and (45) and recognizing that $|V_g/c| \ll 1$, the constants B', C', D', C_1, C_2, D_1 and D_2 can be simplified. Of particular interest is

$$\frac{B'}{A'} \cong \frac{1+i[1-(2\omega'\eta)^{\frac{1}{2}}/c]}{1+i[1+(2\omega'\eta)^{\frac{1}{2}}/c]}, \tag{46}$$

which is the reflexion coefficient in the shock frame. It is easy to show that the magnitude of the reflexion coefficient is minimized at $\omega' = c^2/\eta$. Under this condition, provided that $|\omega'/V_g| \gg |V_g/\eta|$ and $|V_g/c| \ll 1$, the reflexion coefficient becomes

$$(B'/A')_{\min} \cong \frac{1+i(1-\sqrt{2})}{1+i(1+\sqrt{2})} = 0{\cdot}304\,e^{-i96^\circ}. \tag{47}$$

The angular frequencies can then be written

$$\omega_{T1} \cong \omega'\{1 - V_s/V_g\}, \quad \omega_{T2} \cong \omega'\{1 - V_s/(2\eta\omega')^{\frac{1}{2}}\}. \tag{48}$$

As an example we choose $V_g = 10^5\,\text{m/sec}$, $\eta = 10^4\,\text{m}^2/\text{sec}$ and $V_s = 4V_g$ and as long as $\omega' \gg 10^5$ the conditions on (47) are satisfied. Then to minimize the magnitude of the reflexion coefficient $\omega' \cong 9 \times 10^{12}$ which is a frequency of $1{\cdot}43 \times 10^{12}$ c/s. In this particular case $\omega_{T1} \cong -3\omega'$ or $\omega_{T1} \cong -2{\cdot}70 \times 10^{13}$ and $\omega_{T2} \cong (9 - 0{\cdot}0002) \times 10^{12}$. Hence we have one transmitted wave in the ionized medium, ω_{T1}, which undergoes a drastic frequency change while the other frequency ω_{T2} is lowered slightly. Furthermore, the slow wave ω_{T1} is not attenuated nearly as much as the ω_{T2} wave.

The phase velocity (in the laboratory frame) of the backward slow wave is ω_{T1}/γ_1 which is $(V_g - V_s)$. Since $(V_g - V_s)$ is a negative quantity the wave actually travels backward in the negative x-direction. In fact, this velocity is precisely the velocity of the ionized gas in the laboratory frame.

6. Conclusions

The interaction of transverse electromagnetic plane waves with a moving ionizing shock wave are investigated. General expressions for the electromagnetic fields, gas velocities and propagation constants are derived. However, the dispersion equation is not solved in general but rather for the case when $|\omega'/V_g| \gg |V_g/\eta|$, which is the situation in most cases of practical interest. Under this condition it is possible to minimize the magnitude of the electromagnetic reflexion coefficient. For example this minimum occurs at a frequency of $1{\cdot}43 \times 10^{12}$ c/s when $\eta = 10^4\,\mathrm{m}^2/\mathrm{sec}$. If it is desirable to propagate through the shock at a lower frequency with minimum reflexion, then the conductivity of the ionized gas must be decreased. For example, at $\omega' = 62{\cdot}8 \times 10^9$ corresponding to a frequency of 10^{10} c/s, to produce minimum reflexion the conductivity must be adjusted such that $\eta = 1{\cdot}43 \times 10^6$ or $\sigma = 0{\cdot}555$ mho/m.

Two forward travelling transmitted waves exist in the ionized gas, a fast wave and a slow wave, in the shock frame of reference. In the gas frame of reference the slow wave is a standing wave. In the laboratory frame of reference the slow wave actually appears as a slow backward wave travelling in the negative x-direction with a phase velocity equal to the absolute velocity of the ionized gas.

Exact expressions, including relativistic effects, were used for all equations (except the equation of motion. Hence the results should be valid for relativistic values of the velocities V_s and V_g so long as $|V_z/c|^2 \ll 1$. However, the solution of the dispersion equation is only approximate so that the final results should be applied only for situations in which $|V_s/c|$ and $|V_g/c| \ll 1$.

Although the calculations have been applied specifically to a moving shock wave, the results should hold for any free plasma surface moving with velocity V_s (which might be positive or negative) and any arbitrary V_g (positive or negative). In the case where $V_g = 0$ (the plasma moving as a whole), the solutions correspond to those obtained by Fainberg & Tkalich (1959) for an electron gas.

The most striking result of these calculations is the possibility of producing new transmitted transverse waves in a plasma with a drastically different frequency from that of the incident wave, and which do not attenuate so rapidly as the ordinary wave with a frequency near that of the incident wave. From equations (44) and (45) the attenuation constant (in the shock frame) of the ω_{T1} wave is $(a^2/\eta V_g)$ but for ω_{T2} it is $(\omega'/2\eta)^{\frac{1}{2}}$. In general $(a^2/\eta V_g) \ll (\omega'/2\eta)^{\frac{1}{2}}$. For instance in the example discussed in the previous section, with $\omega' = 9 \times 10^{12}$ and $\eta = 10^4\,\mathrm{m}^2/\mathrm{sec}$, $(\omega'/2\eta)^{\frac{1}{2}} = 2{\cdot}1 \times 10^4$. $(a^2/\eta V_g)$ depends on the Alfvèn speed and hence may vary considerably depending on the magnitude of the applied magnetic field B_0 and the density of the gas, but could under conditions easily achieved in practice be of order unity.

Several practical applications, aside from diagnostics, come to mind and may warrant further consideration: transmission through plasma sheaths and laser- or maser-beam frequency multiplication may be mentioned.

7. Extension to the more general problem

In the above calculation and discussion, it has been assumed that the plasma surface was generated by a shock wave and that $|V_s| > |V_g|$. The numerical values of V_s and V_g were taken positive in the direction defined in the text.

However, the results of the calculations are valid for any general numerical values of V_s or V_g, either of which may be positive or negative. A negative value of V_s (V_s is positive in the negative x-direction) indicates that the radiation source (laboratory frame) is receding from the plasma surface and very large positive values of V_s can be achieved if the source is made to approach the free surface. V_g is the velocity of the ionized gas relative to the shock front or plasma surface, measured positive in the positive x-direction. A negative value of V_g indicates that the plasma free surface or front de-ionizes the gas as it flows into the front. Furthermore, in general, $|V_s|$ may be larger than or less than $|V_g|$.

The effects of changing the numerical values of V_s and V_g are the following. The Doppler shift is positive or negative depending on whether V_s is positive or negative. To the order of the approximations made in solving the dispersion equation, the slow wave in the ionized gas is a backward or forward wave (in the positive-x sense) depending respectively on whether the numerical value $(V_s - V_g)$ is positive or negative since the phase velocity of the slow wave is

$$\omega_{T1}/\gamma_1 = (V_s - V_g).$$

The frequency ω_{T1} (in terms of ω') is increased or decreased as shown by equation (48).

The work underlying this paper was supported by a grant from the National Science Foundation.

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